МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ

НАЦІОНАЛЬНИЙ ТЕХНІЧНИЙ УНІВЕРСИТЕТ  
«ХАРКІВСЬКИЙ ПОЛІТЕХНІЧНИЙ ІНСТИТУТ»

Кафедра «Системний аналіз та управління»

Розрахункове завдання   
з дисципліни «Дискретна математика»

(Варіант 24)

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Харків 2017



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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Матрица смежности для неориентированого графа | | | | | | | | | | | | | | |
|  | Х1 | Х2 | Х3 | Х4 | Х5 | Х6 | Х7 | Х8 | Х9 | Х10 | Х11 | Х12 | Х13 | Х14 |
| Х1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х2 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х3 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х4 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Х5 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| Х6 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| Х7 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| Х8 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| Х9 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Х10 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| Х11 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| Х12 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| Х13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 |
| Х14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |

**Завдання 1:**Составить матрицы смежности и инцидентности.

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| Матрица смежности для ориентированого графа | | | | | | | | | | | | | | |
|  | Х1 | Х2 | Х3 | Х4 | Х5 | Х6 | Х7 | Х8 | Х9 | Х10 | Х11 | Х12 | Х13 | Х14 |
| Х1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х2 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х3 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х4 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Х5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| Х6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| Х7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| Х8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| Х9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Х10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| Х11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| Х12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Х13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Х14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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| Матрица инцидентности для неориентированого графа | | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | а1 | а2 | а3 | а4 | а5 | а6 | а7 | а8 | а9 | а10 | а11 | а12 | а13 | а14 | а15 | а16 | а17 | а18 | а19 | а20 | а21 | а22 | а23 | а24 | а25 |
| Х1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х2 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х3 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х4 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х5 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х7 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х8 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| Х9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Х10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Х11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| Х12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| Х13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| Х14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |

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| Матрица инцидентности для ориентированого графа | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | а1 | а2 | а3 | а4 | а5 | а6 | а7 | а8 | а9 | а10 | а11 | а12 | а13 | а14 | а15 | а16 | а17 | а18 | а19 | а20 | а21 | а22 | а23 | а24 | а25 |
| Х1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х2 | -1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х3 | 0 | -1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х4 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х5 | 0 | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х7 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | -1 | -1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х8 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| Х9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Х10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Х11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| Х12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 1 | 0 | 0 | 0 | 0 | 1 |
| Х13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 | -1 | 1 | 0 |
| Х14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -1 |

**Завдання 2:** Определить количество путей в графе длинной 3.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Матрица смежности для ориентированого графа | | | | | | | | | | | | | | |
|  | Х1 | Х2 | Х3 | Х4 | Х5 | Х6 | Х7 | Х8 | Х9 | Х10 | Х11 | Х12 | Х13 | Х14 |
| Х1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х2 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х3 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х4 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Х5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| Х6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| Х7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| Х8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| Х9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Х10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| Х11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| Х12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Х13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Х14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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| Количество путей в графе длинной 2 | | | | | | | | | | | | | | |
|  | Х1 | Х2 | Х3 | Х4 | Х5 | Х6 | Х7 | Х8 | Х9 | Х10 | Х11 | Х12 | Х13 | Х14 |
| Х1 | 0 | 0 | 0 | 1 | 2 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 0 | 1 | 0 | 0 |
| Х3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 2 | 2 | 0 | 1 | 0 |
| Х4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 2 | 0 | 0 |
| Х5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 2 | 0 |
| Х6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| Х7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| Х8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 |
| Х9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Х10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Х11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Х12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Количество путей в графе длинной 3 | | | | | | | | | | | | | | |
|  | Х1 | Х2 | Х3 | Х4 | Х5 | Х6 | Х7 | Х8 | Х9 | Х10 | Х11 | Х12 | Х13 | Х14 |
| Х1 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 4 | 2 | 1 | 1 | 0 |
| Х2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 3 | 3 | 1 |
| Х3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 1 | 5 | 1 |
| Х4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 |
| Х5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 |
| Х6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 |
| Х7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| Х8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| Х9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**Завдання 3:** Построить конденсацию графа

R(Х1)={Х1}ꓴ{Х2,Х3}ꓴ{Х4,Х5,Х7,Х6,Х8}ꓴ{Х7,Х9,Х8,Х10,Х12,Х11,Х13}ꓴ{Х9,Х10,Х12,Х11,Х13,Х14}ꓴ{Х12,Х13,Х14}ꓴ{Х14}ꓴ{Ø}ꓴ{Х1,Х2,Х3,Х4,Х5,Х6,Х7,Х8,Х9,Х10,Х11,Х12,Х13,Х14};

R(Х2)={Х2}ꓴ{Х4,Х5,Х7}ꓴ{Х7,Х9,Х8,Х10,Х12}ꓴ{Х9,Х10,Х12,Х11,Х13,Х14}ꓴ{Х12,Х13,Х14}ꓴ{Х14}ꓴ{Ø}ꓴ{Х2,Х4,Х5,Х7,Х8,Х9,Х10,Х11,Х12,Х13,Х14};

R(Х3)={Х3}ꓴ{Х5,Х6,Х8}ꓴ{Х8,Х10,Х7,Х11,Х13}ꓴ{Х10,Х11,Х13,Х9,Х12,Х14}ꓴ{Х13,Х12,Х14}ꓴ{Х14}ꓴ{Ø}ꓴ{Х3,Х5,Х6,Х7,Х8,Х9,Х10,Х11,Х12,Х13,Х14};

R(Х4)={Х4}ꓴ{Х7,Х9}ꓴ{Х9,Х10,Х12}ꓴ{Х12,Х13,Х14}ꓴ{Х14}ꓴ{Ø}ꓴ{Х4,Х7,Х9,Х10,Х12,Х13,Х14};

R(Х5)={Х5}ꓴ{Х8,Х10}ꓴ{Х10,Х11,Х13}ꓴ{Х13,Х14}ꓴ{Х14}ꓴ{Ø}ꓴ{Х5,Х8,Х10,Х11,Х13,Х14};

R(Х6)={Х6}ꓴ{Х7,Х11}ꓴ{Х9,Х10,Х12,Х13}ꓴ{Х12,Х13,Х14}ꓴ{Х14}ꓴ{Ø}ꓴ{Х6,Х7,Х9,Х10,Х12,Х13,Х14};

R(Х7)={Х7}ꓴ{Х9,Х10,Х12}ꓴ{Х12,Х13,Х14}ꓴ{Х14}ꓴ{Ø}ꓴ{Х7,Х9,Х10,Х12,Х13,Х14};

R(Х8)={Х8}ꓴ{Х10,Х11,Х13}ꓴ{Х13,Х14}ꓴ{Х14}ꓴ{Ø}ꓴ{Х8,Х10,Х11,Х13,Х14};

R(Х9)={Х9}ꓴ{Х12}ꓴ{Х14}ꓴ{Ø}ꓴ{Х9,Х12,Х14};

R(Х10)={Х10}ꓴ{Х13}ꓴ{Х14}ꓴ{Ø}ꓴ{Х10,Х13,Х14};

R(Х11)={Х11}ꓴ{Х13}ꓴ{Х14}ꓴ{Ø}ꓴ{Х11,Х13,Х14};

R(Х12)={Х12}ꓴ{Х14}ꓴ{Ø}ꓴ{Х12,Х14};

R(Х13)={Х13}ꓴ{Х14}ꓴ{Ø}ꓴ{Х13,Х14};

R(Х14)={Х14}ꓴ{Ø}ꓴ{Х14};

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| Матрица достижимости R= | | | | | | | | | | | | | | |
|  | Х1 | Х2 | Х3 | Х4 | Х5 | Х6 | Х7 | Х8 | Х9 | Х10 | Х11 | Х12 | Х13 | Х14 |
| Х1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Х2 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Х3 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Х4 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| Х5 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| Х6 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| Х7 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| Х8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| Х9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| Х10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| Х11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| Х12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| Х13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| Х14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Матрица контрдостижимости Q=RT= | | | | | | | | | | | | | | |
|  | Х1 | Х2 | Х3 | Х4 | Х5 | Х6 | Х7 | Х8 | Х9 | Х10 | Х11 | Х12 | Х13 | Х14 |
| Х1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х2 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х3 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х4 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х5 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х6 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х7 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х8 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х9 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Х10 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| Х11 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| Х12 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| Х13 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| Х14 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Q\*R= | | | | | | | | | | | | | | |
|  | Х1 | Х2 | Х3 | Х4 | Х5 | Х6 | Х7 | Х8 | Х9 | Х10 | Х11 | Х12 | Х13 | Х14 |
| Х1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х3 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х4 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х5 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х6 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х7 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Х10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| Х11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| Х12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Х13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| Х14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

Конденсация графа совпадает с самим графом.

Х1\* = {Х1}

Х2\* = {Х2}

Х3\* = {Х3}

Х4\* = {Х4}

Х5\* = {Х5}

Х6\* = {Х6}

Х7\* = {Х7}

Х8\* = {Х8}

Х9\* = {Х9}

Х10\* = {Х10}

Х11\* = {Х11}

Х12\* = {Х12}

Х13\* = {Х13}

Х14\* = {Х14}

База графа: В={X1} Антибаза графа: ={X14}

База конденсации: В\*={X1\*} Антибаза конденсации: \*={X14\*}

**Задание 4:** Найти кратчайшее расстояние методом Дейсктры

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Матрица смежности для ориентированого графа | | | | | | | | | | | | | | |
|  | Х1 | Х2 | Х3 | Х4 | Х5 | Х6 | Х7 | Х8 | Х9 | Х10 | Х11 | Х12 | Х13 | Х14 |
| Х1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х2 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х3 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| Х4 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Х5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| Х6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| Х7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| Х8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| Х9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Х10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| Х11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| Х12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Х13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Х14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

І этап

d(Х1) = 0; d(Х2)=… d(Х14)= ∞;

∀ Х ≠ Х1

II этап

* y = Х1;

d(Х2) = min{d(Х2),d(y)+a(y,Х2)} = min{∞;0+5}=5;

d(Х3) = min{d(Х3),d(y)+a(y,Х3)} = min{∞;0+5}=5;

* y = Х2;

d(Х4) = min{d(Х4),d(y)+a(y,Х4)} = min{∞;5+2}=7;

d(Х5) = min{d(Х5),d(y)+a(y,Х5)} = min{∞;5+2}=7;

d(Х3) = 5;

* y = Х3;

d(Х5) = min{d(Х5),d(y)+a(y,Х5)} = min{7;5+1}=6;

d(Х6) = min{d(Х6),d(y)+a(y,Х6)} = min{∞;5+3}=8;

d(Х8) = min{d(Х8),d(y)+a(y,Х8)} = min{∞;5+2}=7;

d(Х4) = 7;

d(Х7) = 7;

* y = Х5;

d(Х8) = min{7;6+1}=7;

d(Х10) = min{∞;6+1}=7;

d(Х4) = 7;

d(Х6) = 8;

d(Х7) = 7;

* y = Х4;

d(Х7) = min{7;7+2}=7;

d(Х9) = min{∞;7+3}=10;

d(Х6) = 8;

d(Х8) = 7;

d(Х10) = 7;

* y = Х7;

d(Х9) = min{10;7+1}=8;

d(Х10) = min{7;7+1}=7;

d(Х12) = min{∞;7+1}=8;

d(Х6) = 8;

d(Х8) = 7;

* y = Х8;

d(Х10) = min{7;7+2}=7;

d(Х11) = min{∞;7+2}=9;

d(Х13) = min{∞;7+2}=9;

d(Х9)= 8;

d(12) = 8;

d(Х6) = 8;

* y = Х10;

d(Х13) = min{9;7+3}=10;

d(Х6) = 8;

d(Х9) = 8;

d(Х11) = 9;

d(Х12) = 8;

d(Х13) = 9;

* y = Х6;

d(Х7) = min{7,8+1}=7;

d(Х11) = min{9,8+2}=9;

d(Х9) = 8;

d(Х12) = 8;

d(Х13) = 9;

* y = Х7;

d(Х9) = min{8,7+1}=8;

d(Х10) = min{7,7+1}=7;

d(Х12) = min{8,7+1}=8;

d(Х11) = 9;

d(Х13) = 9;

* y = Х10;

d(Х13) = min{9;7+3}=9;

d(Х9) = 8;

d(Х12) = 8;

d(Х11) = 9;

* y = Х9;

d(Х12) = min{8,8+4}=8;

d(Х11) = 9;

d(Х13) = 9;

* y = Х12;

d(Х14) = min{∞,8+4}=12;

d(Х11) = 9;

d(Х13) = 9;

* y = Х11;

d(Х13) = min{9,9+2}=9;

d(Х14) = 12;

* y = Х13;

d(Х14) = min{12,9+7}=12;

**Задание 6:** Найти кратчайшее расстояние методом Данцига

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Матрица смежности для ориентированого графа | | | | | | | | | | | | | | |
|  | Х1 | Х2 | Х3 | Х4 | Х5 | Х6 | Х7 | Х8 | Х9 | Х10 | Х11 | Х12 | Х13 | Х14 |
| Х1 | 0 | 5 | 5 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| Х2 | ∞ | 0 | ∞ | 2 | 2 | ∞ | 2 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| Х3 | ∞ | ∞ | 0 | ∞ | 1 | 3 | ∞ | 2 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| Х4 | ∞ | ∞ | ∞ | 0 | ∞ | ∞ | 2 | ∞ | 3 | ∞ | ∞ | ∞ | ∞ | ∞ |
| Х5 | ∞ | ∞ | ∞ | ∞ | 0 | ∞ | ∞ | 1 | ∞ | 1 | ∞ | ∞ | ∞ | ∞ |
| Х6 | ∞ | ∞ | ∞ | ∞ | ∞ | 0 | 1 | ∞ | ∞ | ∞ | 2 | ∞ | ∞ | ∞ |
| Х7 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 0 | ∞ | 1 | 1 | ∞ | 1 |  | ∞ |
| Х8 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 0 | ∞ | 2 | 2 | ∞ | 2 | ∞ |
| Х9 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 0 | ∞ | ∞ | 4 | ∞ | ∞ |
| Х10 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 0 | ∞ | ∞ | 3 | ∞ |
| Х11 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 0 | ∞ | 2 | ∞ |
| Х12 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 0 | ∞ | 4 |
| Х13 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 0 | 7 |
| Х14 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | 0 |

D1 =[d111]=[0];

D2=[ d211 d212 ] =[0 5] = [ - (1,2) ]

d221  d222 ∞ 0 - -

d212 = min{ d111+ d012}=min{0+5}=5;

d221 = min{ d021+ d111}=min{∞+0}=∞;

D3=[ d311 d312 d313 ] =[0 5 5] = [ - (1,2) (1,3) ]

d321  d322 d323 ∞ 0 ∞ - - -

d331  d332  d333 ∞ ∞ 0 - - -

d313 = min{ d211+ d013, d212+ d023}=min{0+5,5+∞}=5;

d323 = min{ d221+ d013, d222+ d023}=min{∞+5,0+∞}=∞;

d331 = min{ d031+ d211, d032+ d221}=min{∞+0, ∞+∞}=∞;

d332 = min{ d031+ d212, d032+ d222}=min{∞+5,∞+0}=∞;

D4=[ d411 d412 d413 d414] =[ 0 5 5 ∞ ] = [ - (1,2) (1,3) - ]

d421  d422 d423 d424 ∞ 0 ∞ 2 - - - (2,4)

d431  d432  d433 d413 ∞ ∞ 0 ∞ - - - -

d441  d442  d443 d443 ∞ ∞ ∞ 0 - - - -

d414 = min{ d311+ d014, d312+ d024, d313+ d034}=min{0+∞,5+∞,5+∞}=∞;

d424 = min{ d321+ d014, d322+ d024, d323+ d034}=min{∞+∞,0+2,∞+∞}=2;

d434 = min{ d321+ d014, d332+ d024, d333+ d034}=min{∞+∞,∞+2,0+∞}=∞;

d441 = min{ d041+ d311, d042+ d321, d043+ d331}=min{∞+0, ∞+∞,∞+∞}=∞;

d442 = min{ d041+ d312, d042+ d322, d043+ d333}=min{∞+5, ∞+0,∞+∞}=∞;

d443 = min{ d041+ d313, d042+ d323, d043+ d333}=min{∞+∞, ∞+∞,∞+0}=∞;

D5=[ d511 d512 d513 d514 d515] =[ 0 5 5 ∞ 6 ] = [ - (1,2) (1,3) - (1,3)(3,5)]

d521 d522 d523 d524 d525 ∞ 0 ∞ 2 2 - - - (2,4) (2,5)

d531 d532 d533 d534 d535 ∞ ∞ 0 ∞ 1 - - - - (3,5)

d541 d542 d543 d544 d545 ∞ ∞ ∞ 0 ∞ - - - - -

d551 d552 d553 d554 d555 ∞ ∞ ∞ ∞ 0 - - - - -

d515 = min{ d411+ d015, d412+ d025, d413+ d035, d414+ d045}=min{0+∞,5+2,5+1, ∞+∞}=6;

d525 = min{ d421+ d015, d422+ d025, d423+ d035, d424+ d045}=min{∞+∞,0+2,∞+1, 2+∞}=2;

d535 = min{ d431+ d015, d432+ d025, d433+ d035, d434+ d045}=min{∞+∞,∞+2,0+1, ∞+∞}=1;

d545 = min{ d441+ d015, d442+ d025, d443+ d035, d444+ d045}=min{∞+∞,∞+2, ∞+1, 0+∞}=∞;

d551 = d552 = d553 = d554 =∞;

D6=[ d611 d612 d613 d614 d615 d616]=[0 5 5 ∞ 6 8] = [ - (1,2) (1,3) - (1,3)(3,5) (1,3)(3,6)]

d621 d622 d623 d624 d625 d626 ∞ 0 ∞ 2 2 ∞ - - - (2,4) (2,5) -

d631 d632 d633 d634 d635 d636 ∞ ∞ 0 ∞ 1 3 - - - - (3,5) (3,6)

d641 d642 d643 d644 d645 d646 ∞ ∞ ∞ 0 ∞ ∞ - - - - - -

d651 d652 d653 d654 d655 d656 ∞ ∞ ∞ ∞ 0 ∞ - - - - - -

d661 d662 d663 d664 d665 d666 ∞ ∞ ∞ ∞ ∞ 0 - - - - - -

d616 = min{ d511+ d016, d512+ d026, d513+ d036, d514+ d046, d515+ d056}=min{0+∞,5+∞,5+3, ∞+∞, 6+∞}=8;

d626 = min{ d521+ d016, d522+ d026, d523+ d036, d524+ d046, d525+ d056}=min{∞+∞,0+∞,∞+3, 2+∞, 2+∞}=∞;

d636 = min{ d531+ d016, d532+ d026, d533+ d036, d534+ d046, d535+ d056}=min{∞+∞,∞+∞,0+3, ∞+∞, 1+∞}=3;

d646 = min{ d541+ d016, d542+ d026, d543+ d036, d544+ d046, d545+ d056}= min{∞+∞,∞+∞,∞+∞,0+∞, ∞+∞}=∞;

d656 = min{ d551+ d016, d552+ d026, d553+ d036, d554+ d046, d555+ d056}= min{∞+∞,∞+∞,∞+∞,∞+∞,0+∞}=∞;

d661 = d662 = d663 = d664 = d665 =∞;

D7=[d711 d712 d713 d714 d715 d716 d717]=[0 5 5 ∞ 6 8 7] = [- (1,2) (1,3) - (1,3)(3,5) (1,3)(3,6) (1,2)(2,7)]

d721 d722 d723 d724 d725 d726 d727 ∞ 0 ∞ 2 2 ∞ 2 - - - (2,4) (2,5) - (2,7)

d731 d732 d733 d734 d735 d736 d737 ∞ ∞ 0 ∞ 1 3 4 - - - - (3,5) (3,6) (3,6)(6,7)

d741 d742 d743 d744 d745 d746 d747 ∞ ∞ ∞ 0 ∞ ∞ 2 - - - - - - (4,7)

d751 d752 d755 d754 d755 d756 d757 ∞ ∞ ∞ ∞ 0 ∞ ∞ - - - - - - -

d761 d762 d763 d764 d765 d766 d767 ∞ ∞ ∞ ∞ ∞ 0 1 - - - - - - (6,7)

d771 d772 d773 d774 d775 d776 d777 ∞ ∞ ∞ ∞ ∞ ∞ 0 - - - - - - -

d717 = 7; d727 = 2; d737 = 4; d747 = 2 ; d757 = ∞; d767 = 1;

d771 = d772 = d773 = d774 = d775 = d776 =∞;

D8=[0 5 5 ∞ 6 8 7 7] = [- (1,2) (1,3) - (1,3)(3,5) (1,3)(3,6) (1,2)(2,7) (1,3)(3,5)]

∞ 0 ∞ 2 2 ∞ 2 3 - - - (2,4) (2,5) - (2,7) (2,5)(5,8)

∞ ∞ 0 ∞ 1 3 4 2 - - - - (3,5) (3,6) (3,6)(6,7) (3,8)

∞ ∞ ∞ 0 ∞ ∞ 2 ∞ - - - - - - (4,7) -

∞ ∞ ∞ ∞ 0 ∞ ∞ 1 - - - - - - - (5,8)

∞ ∞ ∞ ∞ ∞ 0 1 ∞ - - - - - - (6,7) -

∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ - - - - - - - -

∞∞ ∞ ∞ ∞ ∞ ∞ 0 - - - - - - - -

d818 = 7; d828 = 3; d838 = 2; d848 =∞ ; d858 = 1; d868 = ∞; d878 = ∞;

d881 = d882 = d883 = d884 = d885 = d886 = d887 =∞;

D9=[0 5 5 ∞ 6 8 7 7 8] = [- (1,2) (1,3) - (1,3)(3,5) (1,3)(3,6) (1,2)(2,7) (1,3)(3,5) (1,7)(7,9)]

∞ 0 ∞ 2 2 ∞ 2 3 3 - - - (2,4) (2,5) - (2,7) (2,5)(5,8) (2,7)(7,9)

∞ ∞ 0 ∞ 1 3 4 2 4 - - - - (3,5) (3,6) (3,6)(6,7) (3,8) (3,7)(7,9)

∞ ∞ ∞ 0 ∞ ∞ 2 ∞ 3 - - - - - - (4,7) - (4,7)(7,9)

∞ ∞ ∞ ∞ 0 ∞ ∞ 1 ∞ - - - - - - - (5,8) -

∞ ∞ ∞ ∞ ∞ 0 1 ∞ 2 - - - - - - (6,7) - (6,7)(7,9)

∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ 1 - - - - - - - - (7,9)

∞∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ - - - - - - - - -

∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 - - - - - - - - -

d919 = 8; d929 = 3; d939 = 4; d949 =3 ; d959 = ∞; d969 = 2; d979 = 1; d989 = ∞;

d991 = d992 = d993 = d994 = d995 = d996 = d997= d998 =∞;

D10=[0 5 5 ∞ 6 8 7 7 8 7]=[- (1,2) (1,3) - (1,3)(3,5) (1,3)(3,6) (1,2)(2,7) (1,3)(3,5) (1,7)(7,9) (1,5)(5,10)]

∞ 0 ∞ 2 2 ∞ 2 3 3 3 - - - (2,4) (2,5) - (2,7) (2,5)(5,8) (2,7)(7,9) (2,5)(5,10)

∞ ∞ 0 ∞ 1 3 4 2 4 2 - - - - (3,5) (3,6) (3,6)(6,7) (3,8) (3,7)(7,9) (3,5)(5,10)

∞ ∞ ∞ 0 ∞ ∞ 2 ∞ 3 3 - - - - - - (4,7) - (4,7)(7,9) (4,7)(7,10)

∞ ∞ ∞ ∞ 0 ∞ ∞ 1 ∞ 3 - - - - - - - (5,8) - (5,7)(7,10)

∞ ∞ ∞ ∞ ∞ 0 1 ∞ 2 2 - - - - - - (6,7) - (6,7)(7,9) (6,7)(7,10)

∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ 1 1 - - - - - - - - (7,9) (6,7)(7,10)

∞∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ 2 - - - - - - - - - (7,10)

∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ - - - - - - - - - (8,10)

∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 - - - - - - - - - -

d10110 = 7; d10210 = 3; d10310 = 2; d10410 =3; d10510 = 3; d10610 = 2; d10710 = 1; d10810 = 2;

d10910 = ∞;

d10101 = d10102 = d10103 = d10104 = d10105 = d10106 = d10107= d10108 = d10109=∞;

D11=[0 5 5 ∞ 6 8 7 7 8 7 9]

∞ 0 ∞ 2 2 ∞ 2 3 3 3 5

∞ ∞ 0 ∞ 1 3 4 2 4 2 4

∞ ∞ ∞ 0 ∞ ∞ 2 ∞ 3 3 ∞

∞ ∞ ∞ ∞ 0 ∞ ∞ 1 ∞ 3 3

∞ ∞ ∞ ∞ ∞ 0 1 ∞ 2 2 ∞

∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ 1 1 ∞

∞∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ 2 2

∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ ∞

∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 ∞

∞ ∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0

[ - (1,2) (1,3) - (1,3)(3,5) (1,3)(3,6) (1,2)(2,7) (1,3)(3,5) (1,7)(7,9) (1,5)(5,10) (1,6)(6,11)]

- - - (2,4) (2,5) - (2,7) (2,5)(5,8) (2,7)(7,9) (2,5)(5,10) (2,8)(8,11)

- - - - (3,5) (3,6) (3,6)(6,7) (3,8) (3,7)(7,9) (3,5)(5,10) (3,8)(8,11)

- - - - - - (4,7) - (4,7)(7,9) (4,7)(7,10) -

- - - - - - - (5,8) - (5,7)(7,10) (5,8)(8,11)

- - - - - - (6,7) - (6,7)(7,9) (6,7)(7,10) -

- - - - - - - - (7,9) (6,7)(7,10) -

- - - - - - - - - (7,10) (8,11)

- - - - - - - - - (8,10) -

- - - - - - - - - - -

- - - - - - - - - - -

d11111 = 9; d11211 = 5; d11311 = 4; d11411 =∞; d11511 = 3; d11611 = ∞; d11711 = ∞; d11811 = 2;

d11911 = ∞; d111011 = ∞;

d11111 = d11112 = d11113 = d11114 = d11115 = d11116 = d11117= d11118 = d11119= d111110=∞;

D12=[0 5 5 ∞ 6 8 7 7 8 7 9 8]

∞ 0 ∞ 2 2 ∞ 2 3 3 3 5 3

∞ ∞ 0 ∞ 1 3 4 2 4 2 4 5

∞ ∞ ∞ 0 ∞ ∞ 2 ∞ 3 3 ∞ 3

∞ ∞ ∞ ∞ 0 ∞ ∞ 1 ∞ 3 3 ∞

∞ ∞ ∞ ∞ ∞ 0 1 ∞ 2 2 ∞ 2

∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ 1 1 ∞ 1

∞∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ 2 2 ∞

∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ ∞ 4

∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ ∞

∞ ∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 ∞

∞ ∞ ∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0

[ - (1,2) (1,3) - (1,3)(3,5) (1,3)(3,6) (1,2)(2,7) (1,3)(3,5) (1,7)(7,9) (1,5)(5,10) (1,6)(6,11) (1,7)(7,12)]

- - - (2,4) (2,5) - (2,7) (2,5)(5,8) (2,7)(7,9) (2,5)(5,10) (2,8)(8,11) (2,7)(7,12)

- - - - (3,5) (3,6) (3,6)(6,7) (3,8) (3,7)(7,9) (3,5)(5,10) (3,8)(8,11) (3,7)(7,12)

- - - - - - (4,7) - (4,7)(7,9) (4,7)(7,10) - (4,7)(7,12)

- - - - - - - (5,8) - (5,7)(7,10) (5,8)(8,11) -

- - - - - - (6,7) - (6,7)(7,9) (6,7)(7,10) - (6,7)(7,12)

- - - - - - - - (7,9) (6,7)(7,10) - (7,12)

- - - - - - - - - (7,10) (8,11) -

- - - - - - - - - (8,10) - (9,12)

- - - - - - - - - - - -

- - - - - - - - - - - -

- - - - - - - - - - - -

d12121 = 8; d12212 = 3; d12312 = 5; d12412 =3; d12512 = ∞; d12612 = 2; d12712 = 1; d12812 = ∞;

d12912 = 4; d121012 = ∞; d121112 = ∞;

d12121 = d12122 = d12123 = d12124 = d12125 = d12126 = d12127= d12128 = d12129= d121210= d121211=∞;

D13=[0 5 5 ∞ 6 8 7 7 8 7 9 8 9]

∞ 0 ∞ 2 2 ∞ 2 3 3 3 5 3 5

∞ ∞ 0 ∞ 1 3 4 2 4 2 4 5 4

∞ ∞ ∞ 0 ∞ ∞ 2 ∞ 3 3 ∞ 3 6

∞ ∞ ∞ ∞ 0 ∞ ∞ 1 ∞ 3 3 ∞ 3

∞ ∞ ∞ ∞ ∞ 0 1 ∞ 2 2 ∞ 2 5

∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ 1 1 ∞ 1 4

∞∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ 2 2 ∞ 2

∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ ∞ 4 ∞

∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ ∞ 3

∞ ∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ 2

∞ ∞ ∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 ∞

∞ ∞ ∞ ∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0

[ - (1,2) (1,3) - (1,3)(3,5) (1,3)(3,6) (1,2)(2,7) (1,3)(3,5) (1,7)(7,9) (1,5)(5,10) (1,6)(6,11) (1,7)(7,12) (1,8)(8,13)]

- - - (2,4) (2,5) - (2,7) (2,5)(5,8) (2,7)(7,9) (2,5)(5,10) (2,8)(8,11) (2,7)(7,12) (2,8)(8,13)

- - - - (3,5) (3,6) (3,6)(6,7) (3,8) (3,7)(7,9) (3,5)(5,10) (3,8)(8,11) (3,7)(7,12) (3,8)(8,13)

- - - - - - (4,7) - (4,7)(7,9) (4,7)(7,10) - (4,7)(7,12) (4,8)(8,13)

- - - - - - - (5,8) - (5,7)(7,10) (5,8)(8,11) - -

- - - - - - (6,7) - (6,7)(7,9) (6,7)(7,10) - (6,7)(7,12) (9,10)(10,13)

- - - - - - - - (7,9) (6,7)(7,10) - (7,12) (7,8)(8,13)

- - - - - - - - - (7,10) (8,11) - (8,13)

- - - - - - - - - (8,10) - (9,12) -

- - - - - - - - - - - - (10,13)

- - - - - - - - - - - - (11,13)

- - - - - - - - - - - - -

- - - - - - - - - - - - -

d13113 = 9; d13213 = 5; d13313 = 4; d13413 =6; d13513 = 3; d13613 = 5; d13713 = 4; d13813 = 2;

d13913 = ∞; d131013 = 3; d131113 =2; d131113 = ∞;

d13131 = d13132 = d13133 = d13134 = d13135 = d13136 = d13137= d13138 = d13139= d131310= d131311= =d131312=∞;

D14=[0 5 5 ∞ 6 8 7 7 8 7 9 8 9 12]

∞ 0 ∞ 2 2 ∞ 2 3 3 3 5 3 5 7

∞ ∞ 0 ∞ 1 3 4 2 4 2 4 5 4 9

∞ ∞ ∞ 0 ∞ ∞ 2 ∞ 3 3 ∞ 3 6 7

∞ ∞ ∞ ∞ 0 ∞ ∞ 1 ∞ 3 3 ∞ 3 10

∞ ∞ ∞ ∞ ∞ 0 1 ∞ 2 2 ∞ 2 5 6

∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ 1 1 ∞ 1 4 5

∞∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ 2 2 ∞ 2 9

∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ ∞ 4 ∞ 8

∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ ∞ 3 10

∞ ∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ 2 9

∞ ∞ ∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 ∞ 4

∞ ∞ ∞ ∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0 7

∞ ∞ ∞ ∞ ∞ ∞∞ ∞ ∞ ∞ ∞ ∞ ∞ 0

[ - (1,2) (1,3) - (1,3)(3,5) (1,3)(3,6) (1,2)(2,7) (1,3)(3,5) (1,7)(7,9) (1,5)(5,10) (1,6)(6,11) (1,7)(7,12) (1,8)(8,13) (1,12)(12,14)]

- - - (2,4) (2,5) - (2,7) (2,5)(5,8) (2,7)(7,9) (2,5)(5,10) (2,8)(8,11) (2,7)(7,12) (2,8)(8,13) (2,12)(12,14)

- - - - (3,5) (3,6) (3,6)(6,7) (3,8) (3,7)(7,9) (3,5)(5,10) (3,8)(8,11) (3,7)(7,12) (3,8)(8,13) (3,12)(12,14)

- - - - - - (4,7) - (4,7)(7,9) (4,7)(7,10) - (4,7)(7,12) (4,8)(8,13) (4,12)(12,14)

- - - - - - - (5,8) - (5,7)(7,10) (5,8)(8,11) - - (5,13)(13,14)

- - - - - - (6,7) - (6,7)(7,9) (6,7)(7,10) - (6,7)(7,12) (9,10)(10,13) (6,12)(12,14)

- - - - - - - - (7,9) (6,7)(7,10) - (7,12) (7,8)(8,13) (7,12)(12,14)

- - - - - - - - - (7,10) (8,11) - (8,13) (8,13)(13,14)

- - - - - - - - - (8,10) - (9,12) - (9,12)(12,14)

- - - - - - - - - - - - (10,13) (10,13)(13,14)

- - - - - - - - - - - - (11,13) (11,13)(13,14)

- - - - - - - - - - - - - (12,14)

- - - - - - - - - - - - - (13,14)

- - - - - - - - - - - - - -

d14114 = 12; d14214 = 7; d14314 = 9; d14414 =7; d14514 = 10; d14614 = 6; d14714 = 5; d14814 = 9; d14914 = 8; d141014 = 10; d141114 =9; d141114 = 4; d141114 = 7;

d14141 = d14142 = d14143 = d14144 = d14145 = d14146 = d14147= d14148 = d14149= d141410= d141411= =d141412=d141413=∞;

**Завдання 7:** Найти максимальный поток.

Шаг 1

f(x,y)=0, ∀(x,y);

Распределим дуги по множествам

f(x1, x2) = 0 => с(x1, x2)- f(x1, x2)= с(x1, x2)=5 є I

f(x1, x3) = 0 => c(x1, x3)- f(x1, x3)= c(x1, x3)=5 є I

f(x2, x4) = 0 => c(x2, x4) - f(x2, x4)= c(x2, x4) =2 є I

f(x2, x5) = 0 => c(x2, x5)=2 є I

f(x2, x7) = 0 => c(x2, x7) =2 є I

f(x3, x5) = 0 => c(x3, x5) =1 є I

f(x3, x6) = 0 => c(x3, x6) =3 є I

f(x3, x8) = 0 => c(x3, x8) =2 є I

f(x4, x7) = 0 => c(x4, x7) =2 є I

f(x4, x9) = 0 => c(x4, x9) =3 є I

f(x5, x8) = 0 => c(x5, x8) = 1 є I

f(x5, x10) = 0 => c(x5, x10)= 1 є I

f(x6, x7) = 0 => c(x6, x7) =1 є I

f(x6, x11) = 0 => c(x6, x11)=2 є I

f(x7, x9) = 0 => c(x7, x9) =1 є I

f(x7, x10) = 0 => c(x7, x10) є I

f(x7, x12) = 0 => c(x7, x12) =1 є I

f(x8, x10) = 0 => c(x8, x10) =2 є I

f(x8, x11) = 0 => c(x8, x11) =2 є I

f(x8, x13) = 0 => c(x8, x13) =2 є I

f(x9, x12) = 0 => c(x9, x12) =4 є I

f(x10, x13) = 0 => c(x10, x13) =3 є I

f(x11, x13) = 0 => c(x11, x13) =2 є I

f(x12, x14) = 0 => c(x12, x14) =4 є I

f(x13, x14) = 0 => c(x13, x14) =7 є I

Шаг 2

Применим алгоритм поиска увеличивающегося пути.

Пусть увеличивающий путь будет (1, 2) (2, 7) (7, 12) (12, 14)

Максимальное увеличение составляет:

min={i(1,2);i(2,7);i(7,12);i(12,14)}=min{5;2;1;4}=1;

f(1, 2) = 0 + 1 = 1

f(2, 7) = 0 + 1 = 1

f(7, 12) = 0 + 1 = 1

f(12, 14) = 0 + 1 = 1

f(1, 2) >=1; c(1,2)=5 ->(1,2) IR

f(2, 7)> = 1; с(2,7)=2 ->(2,7) IR

f(7, 12) >=1; c(7,12)=1 ->(7,12) R

f(12, 14) >= 1; c(12,14)=4; ->(12,14) IR

Шаг 3

Пусть увеличивающий путь будет (1, 2) (2, 4) (4, 9) (9, 12) (12,14)

Максимальное увеличение составляет:

min={i(1, 2);i (2, 4); i(4, 9); i(9, 12);i(12,14)}=min{4;2;3;4;3}=2;

f(1, 2) = 1+2 = 3

f(2, 4) = 0 + 2 = 2

f(4, 9) = 0 + 2 = 2

f(9, 12) = 0 + 2 = 2

f(12, 14) = 1 + 2 = 3

f(1, 2) >=3; c(1,2)=5 ->(1,2) IR

f(2, 4)> = 2; с(2,4)=2 ->(2,4) R

f(4, 9) >=2; c(4,9)=3 ->(4,9) IR

f(9, 12) >= 2; c(9,12)=4; ->(9,12) IR

f(12, 14) >= 3; c(12,14)=4; ->(12,14) IR

Шаг 4



Пусть увеличивающий путь будет (1, 2) (2, 7) (7, 9) (9, 12) (12,14)

Максимальное увеличение составляет:

min={i(1, 2);i (2, 7); i(7, 9); i(9, 12);i(12,14)}=min{2;1;1;2;1}=1;

f(1, 2) = 3+1 = 4

f(2, 7) = 1+1 = 2

f(7, 9) = 0 + 1 = 1

f(9, 12) = 2+1 = 3

f(12, 14) = 2+1 = 3

f(1, 2) >=4; c(1,2)=5 ->(1,2) IR

f(2, 7)> = 2; с(2,7)=2 ->(2,4) R

f(7, 9) >=1; c(7,9)=1 ->(4,9) R

f(9, 12) >= 3; c(9,12)=4; ->(9,12) IR

f(12, 14) >= 3; c(12,14)=4; ->(12,14) IR

Шаг 5

Пусть увеличивающий путь будет (1, 3) (3, 8) (8, 13) (13,14)

Максимальное увеличение составляет:

min={i(1, 3); i(3, 8); i(8, 13); i (13,14)}=min{5;2;2;7}=2;

f(1, 3) = 0+2 = 2

f(3, 8) = 0+2 = 2

f(8, 13) = 0 + 2 = 2

f(13, 14) = 0+2 = 2

f(1, 3) >=2; c(1,3)=5 ->(1,2) IR

f(3, 8)> = 2; с(3,8)=2 ->(2,4) R

f(8, 13) >=2; c(8,13)=2 ->(4,9) R

f(13, 14) >= 2; c(13,14)=7; ->(12,14) IR

Шаг 6

Пусть увеличивающий путь будет (1, 2) (2, 5) (5, 10) (10,13) (13,14)

Максимальное увеличение составляет:

min={i(1, 2); i(2, 5);i (5, 10);i (10,13);i (13,14) }=min{1;2;1;3;5}=1;

f(1, 2) = 4+1 = 5

f(2, 5) = 0+1 = 1

f(5,10) = 0 + 1 = 1

f(10, 13) = 0+1 = 1

f(13, 14) = 2+1 = 3

f(1, 2) >=5; c(1,2)=5 ->(1,2) R

f(2, 5)> = 1; с(2,5)=2 ->(2,4) IR

f(5, 10) >=1; c(5,10)=1 ->(4,9) R

f(10, 13) >=1; c(10,13)=3 ->(4,9) IR

f(13, 14) >= 3; c(13,14)=7; ->(12,14) IR

Шаг 7



Пусть увеличивающий путь будет (1, 3) (3, 6) (6,11) (11, 13) (13,14)

Максимальное увеличение составляет:

min={i(1, 3); i(3, 6);i(6,11); i(11, 13); i (13,14)}=min{3;3;2;2;4}=2;

f(1, 3) = 2+2 = 4

f(3, 6) = 0+2 = 2

f(6, 11) = 0 + 2 = 2

f(11, 13) = 0+2 = 2

f(13, 14) = 3+2 = 5

f(1, 3) >=4; c(1,3)=5 ->(1,2) IR

f(3, 6)> = 2; с(3,6)=3 ->(2,4) IR

f(6, 11) >=2; c(6,11)=2 ->(4,9) R

f(11, 13) >=2; c(11,13)=2 ->(4,9) R

f(13, 14) >= 5; c(13,14)=7; ->(12,14) IR

Шаг 8



Пусть увеличивающий путь будет (1, 3) (3, 6) (6,7) (7,10) (10, 13) (13,14)

Максимальное увеличение составляет:

min={i(1, 3); i(3, 6);i(6,7);i(7,10); i(10, 13); i (13,14)}=min{1;1;1;1;2;2}=1;

f(1, 3) = 4+1 = 5

f(3, 6) = 2+1 = 3

f(6,7) = 0+1 = 1

f(7, 10) = 0 + 1 = 1

f(10, 13) = 1+1 = 2

f(13, 14) = 5+1 = 6

f(1, 3) >=5; c(1,3)=5 ->(1,2) R

f(3, 6)> = 3; с(3,6)=3 ->(2,4) R

f(6, 7)> = 1; с(6,7)=1 ->(2,4) R

f(7, 10) >=1; c(7,10)=1 ->(4,9) R

f(10, 13) >=2; c(10,13)=3 ->(4,9) ІR

f(13, 14) >= 6; c(13,14)=7; ->(12,14) IR

Шаг 9



Алгоритм окончен.

Максимальное увеличение потока 10

Пройденные пути:

(1, 2) (2, 7) (7, 12) (12, 14) -1

(1, 2) (2, 4) (4, 9) (9, 12) (12,14) -2

(1, 2) (2, 7) (7, 9) (9, 12) (12,14) -1

(1, 3) (3, 8) (8, 13) (13,14) -2

(1, 2) (2, 5) (5, 10) (10,13) (13,14) -1

(1, 3) (3, 6) (6,7) (7,10) (10, 13) (13,14) -2

(1, 3) (3, 6) (6,11) (11, 13) (13,14) -1

**Завдання 8:** Найти макимальный поток минимальной стоимости

Полагаем, что все вершинные числа равны 0

Все вершины не окрашены за исключением вершины x1

p(x1) = 0; p(x2) = p(x14) = 0

Сформируем множества I, R, N

p(x2) – p(x1) = 0 ≠ a(x1,x2) = 2, f(x1,x2) = 0 ∈ N

p(x3) – p(x1) = 0 ≠ a(x1,x3) = 3, f(x1,x3) = 0 ∈ N

……………………….

P(x14) – p(x13) = 0 ≠ a(x13,x14) = 5, f(x13,x14) = 0 ∈ N

Все дуги относятся к множеству N, а поэтому применить алгоритм поиска максимального потока нельзя.

Увеличиваем вершинные числа неокрашенных вершин на единицу.

p(x1) = 0; p(x2) = … = p(x14) = 0 + 1 = 1

Повторяем шаг 2. Формируем множества I, R, N

p(x2) – p(x1) =1 ≠ a(x1,x2) = 2, f(x1,x2) = 0 ∈ N

p(x3) – p(x1) = 1 ≠ a(x1,x3) = 3, f(x1,x3) = 0 ∈ N

Максимальный поток найти не удается. Увеличим вершинные числа неокрашенных вершин на единицу.

p(x1) = 0 ; p(x2) = … = p(x14) = 1 + 1 = 2

Повторяем шаг 2. Формируем множества I, R, N

p(x2) – p(x1) =2-0 = a(x1,x2) = 2, f(x1,x2) = 0 ∈ I

p(x3) – p(x1) = 2 -0≠ a(x1,x3) = 3, f(x1,x3) = 0 ∈ N

Выполним результирующее окрашивание.

Максимальный поток найти не удается. Увеличим вершинные числа неокрашенных вершин на единицу.

p(x1) = 0 ; p(x2) = 2; p(x3) = … = p(x14) = 2 + 1 = 3

Повторяем шаг 2. Формируем множества I, R, N

p(x2) – p(x1) =2-0 = a(x1,x2) = 2, f(x1,x2) = 0 ∈ І

p(x3) – p(x1) = 3-0 = a(x1,x3) = 3, f(x1,x3) = 0 ∈ І

p(x4) – p(x2) =3-2=1 = a(x2,x4) = 1, f(x2,x4) = 0 ∈ І

p(x5) – p(x2) =3-2=1 = a(x2,x5) = 1, f(x2,x5) = 0 ∈ І

p(x7) – p(x2) =3-2=1 ≠ a(x2,x7) = 2, f(x2,x7) = 0 ∈ N

Выполним результирующее окрашивание.

Максимальный поток найти не удается. Увеличим вершинные числа неокрашенных вершин на единицу.

p(x1) = 0 ; p(x2) = 2; p(x3) = p(x4)= p(x5) = 3; p(x6) = … = p(x14) = 3 + 1 = 4

Повторяем шаг 2. Формируем множества I, R, N

p(x2) – p(x1) =2-0 = a(x1,x2) = 2, f(x1,x2) = 0 ∈ І

p(x3) – p(x1) = 3-0 = a(x1,x3) = 3, f(x1,x3) = 0 ∈ І

p(x4) – p(x2) =3-2=1 = a(x2,x4) = 1, f(x2,x4) = 0 ∈ І

p(x5) – p(x2) =3-2=1 = a(x2,x5) = 1, f(x2,x5) = 0 ∈ І

p(x7) – p(x2) =4-2=2 = a(x2,x7) = 2, f(x2,x7) = 0 ∈ І

p(x5) – p(x3) =3-3=0 ≠ a(x3,x5) = 1, f(x3,x5) = 0 ∈ N

p(x6) – p(x3) =4-3=1= a(x3,x6) = 1, f(x3,x6) = 0 ∈ І

p(x8) – p(x3) = 4-3=1 ≠ a(x3,x8) = 2, f(x3,x8) = 0 ∈ N

p(x7) – p(x4) = 4-3=1= a(x4,x7) = 1, f(x4,x7) = 0 ∈ І

p(x9) – p(x4) = 4-3=1 = a(x4,x9) = 1, f(x4,x9) = 0 ∈ І

p(x8) – p(x5) =4-3=1 = a(x5,x8) = 1, f(x5,x8) = 0 ∈ I

p(x10) – p(x5) =4-3=1= a(x8,x10) = 1, f(x8,x10) = 0 ∈ I

Выполним результирующее окрашивание.

Максимальный поток найти не удается. Увеличим вершинные числа неокрашенных вершин на единицу.

p(x1) = 0 ; p(x2) = 2; p(x3) = p(x4)=3; p(x5)= p(x6) = p(x7) =p(x9) = p(x10) = 4

p(x8) = p(x11) = … = p(x14) = 4 + 1 = 5

Повторяем шаг 2. Формируем множества I, R, N

p(x2) – p(x1) =2-0 = a(x1,x2) = 2, f(x1,x2) = 0 ∈ І

p(x3) – p(x1) = 3-0 = a(x1,x3) = 3, f(x1,x3) = 0 ∈ І

p(x4) – p(x2) =3-2=1 = a(x2,x4) = 1, f(x2,x4) = 0 ∈ І

p(x5) – p(x2) =3-2=1 = a(x2,x5) = 1, f(x2,x5) = 0 ∈ І

p(x7) – p(x2) =4-2=2 = a(x2,x7) = 2, f(x2,x7) = 0 ∈ І

p(x5) – p(x3) =4-3=1 = a(x3,x5) = 1, f(x3,x5) = 0 ∈ І

p(x6) – p(x3) =4-3=1= a(x3,x6) = 1, f(x3,x6) = 0 ∈ І

p(x8) – p(x3) = 5-3=2 = a(x3,x8) = 2, f(x3,x8) = 0 ∈ І

p(x7) – p(x4) = 4-3=1= a(x4,x7) = 1, f(x4,x7) = 0 ∈ І

p(x9) – p(x4) = 4-3=1 = a(x4,x9) = 1, f(x4,x9) = 0 ∈ І

p(x8) – p(x5) =4-3=1 = a(x5,x8) = 1, f(x5,x8) = 0 ∈ I

p(x10) – p(x5) =4-3=1= a(x8,x10) = 1, f(x8,x10) = 0 ∈ I

p(x9) – p(x7) = 4-4=0 ≠ a(x7,x9) = 1, f(x7,x9) = 0 ∈ N

p(x10) – p(x7) = 4-4=0 ≠ a(x7,x10) = 1, f(x7,x10) = 0 ∈ N

p(x12) – p(x7) =5-4=1 = a(x12,x7) = 1, f(x12,x7) = 0 ∈ I

p(x12) – p(x9) =5-4=1= a(x9,x12) = 1, f(x9,x12) = 0 ∈ I

p(x7) – p(x6) =4-4= 0 ≠ a(x6,x7) = 1, f(x6,x7) = 0 ∈ N

p(x11) – p(x6) = 1= a(x6,x11) = 1, f(x6,x11) = 0 ∈ І

p(x10) – p(x8) =4-5=-1≠ a(x8,x10) = 1, f(x8,x10) = 0 ∈ N

p(x11) – p(x8) = 5-5=0 ≠ a(x8,x11) = 2, f(x8,x11) = 0 ∈ N

p(x13) – p(x8) = 5-5=0 ≠ a(x8,x13) = 2, f(x8,x13) = 0 ∈ N

p(x13) – p(x10) = 1 ≠ a(x10,x13) = 3, f(x10,x13) = 0 ∈ N

Выполним результирующее окрашивание.

Максимальный поток найти не удается. Увеличим вершинные числа неокрашенных вершин на единицу.

p(x1) = 0 ; p(x2) = 2; p(x3) = p(x4)=3; p(x5)= p(x6) = 4

p(x7)= …=p(x10) = 5

p(x11) =…= p(x14) =5+1= 6

Повторяем шаг 2. Формируем множества I, R, N

p(x2) – p(x1) =2-0 = a(x1,x2) = 2, f(x1,x2) = 0 ∈ І

p(x3) – p(x1) = 3-0 = a(x1,x3) = 3, f(x1,x3) = 0 ∈ І

p(x4) – p(x2) =3-2=1 = a(x2,x4) = 1, f(x2,x4) = 0 ∈ І

p(x5) – p(x2) =3-2=1 = a(x2,x5) = 1, f(x2,x5) = 0 ∈ І

p(x7) – p(x2) =4-2=2 = a(x2,x7) = 2, f(x2,x7) = 0 ∈ І

p(x5) – p(x3) =4-3=1 = a(x3,x5) = 1, f(x3,x5) = 0 ∈ І

p(x6) – p(x3) =4-3=1= a(x3,x6) = 1, f(x3,x6) = 0 ∈ І

p(x8) – p(x3) = 5-3=2 = a(x3,x8) = 2, f(x3,x8) = 0 ∈ І

p(x7) – p(x4) = 4-3=1= a(x4,x7) = 1, f(x4,x7) = 0 ∈ І

p(x9) – p(x4) = 4-3=1 = a(x4,x9) = 1, f(x4,x9) = 0 ∈ І

p(x8) – p(x5) =4-3=1 = a(x5,x8) = 1, f(x5,x8) = 0 ∈ I

p(x10) – p(x5) =4-3=1= a(x8,x10) = 1, f(x8,x10) = 0 ∈ I

p(x9) – p(x7) = 5-5=0 ≠ a(x7,x9) = 1, f(x7,x9) = 0 ∈ N

p(x10) – p(x7) = 5-5=0 ≠ a(x7,x10) = 1, f(x7,x10) = 0 ∈ N

p(x12) – p(x7) =5-4=1 = a(x12,x7) = 1, f(x12,x7) = 0 ∈ I

p(x12) – p(x9) =5-4=1= a(x9,x12) = 1, f(x9,x12) = 0 ∈ I

p(x7) – p(x6) =5-4= 1 = a(x6,x7) = 1, f(x6,x7) = 0 ∈ І

p(x11) – p(x6) = 5-4=1= a(x6,x11) = 1, f(x6,x11) = 0 ∈ І

p(x10) – p(x8) =5-5=0≠ a(x8,x10) = 1, f(x8,x10) = 0 ∈ N

p(x11) – p(x8) = 6-5=1 ≠ a(x8,x11) = 2, f(x8,x11) = 0 ∈ N

p(x13) – p(x8) = 6-5=1 ≠ a(x8,x13) = 2, f(x8,x13) = 0 ∈ N

p(x13) – p(x10) = 6-5=1 ≠ a(x10,x13) = 3, f(x10,x13) = 0 ∈ N

p(x13) – p(x11) = 6-6=0 ≠ a(x11,x13) = 2, f(x11,x13) = 0 ∈ N

p(x14) – p(x12) = 6-6=0 ≠ a(x12,x14) = 2, f(x12,x14) = 0 ∈ N

P(x14) – p(x13) = 6-6 ≠ a(x13,x14) = 5, f(x13,x14) = 0 ∈ N

Выполним результирующее окрашивание.



Максимальный поток найти не удается. Увеличим вершинные числа неокрашенных вершин на единицу.

p(x1) = 0 ; p(x2) = 2; p(x3) = p(x4)=3; p(x5)= p(x6) = 4

p(x7)=p(x8) = 5 p(x9) = p(x10)=6;

p(x11) =…= p(x14) =6+1= 7

Повторяем шаг 2. Формируем множества I, R, N

p(x2) – p(x1) =2-0 = a(x1,x2) = 2, f(x1,x2) = 0 ∈ І

p(x3) – p(x1) = 3-0 = a(x1,x3) = 3, f(x1,x3) = 0 ∈ І

p(x4) – p(x2) =3-2=1 = a(x2,x4) = 1, f(x2,x4) = 0 ∈ І

p(x5) – p(x2) =3-2=1 = a(x2,x5) = 1, f(x2,x5) = 0 ∈ І

p(x7) – p(x2) =4-2=2 = a(x2,x7) = 2, f(x2,x7) = 0 ∈ І

p(x5) – p(x3) =4-3=1 = a(x3,x5) = 1, f(x3,x5) = 0 ∈ І

p(x6) – p(x3) =4-3=1= a(x3,x6) = 1, f(x3,x6) = 0 ∈ І

p(x8) – p(x3) = 5-3=2 = a(x3,x8) = 2, f(x3,x8) = 0 ∈ І

p(x7) – p(x4) = 4-3=1= a(x4,x7) = 1, f(x4,x7) = 0 ∈ І

p(x9) – p(x4) = 4-3=1 = a(x4,x9) = 1, f(x4,x9) = 0 ∈ І

p(x8) – p(x5) =4-3=1 = a(x5,x8) = 1, f(x5,x8) = 0 ∈ I

p(x10) – p(x5) =4-3=1= a(x8,x10) = 1, f(x8,x10) = 0 ∈ I

p(x9) – p(x7) = 6-5=1 = a(x7,x9) = 1, f(x7,x9) = 0 ∈ І

p(x10) – p(x7) = 6-5=1 = a(x7,x10) = 1, f(x7,x10) = 0 ∈ І

p(x12) – p(x7) =5-4=1 = a(x12,x7) = 1, f(x12,x7) = 0 ∈ I

p(x12) – p(x9) =5-4=1= a(x9,x12) = 1, f(x9,x12) = 0 ∈ I

p(x7) – p(x6) =5-4= 1 = a(x6,x7) = 1, f(x6,x7) = 0 ∈ І

p(x11) – p(x6) = 5-4=1= a(x6,x11) = 1, f(x6,x11) = 0 ∈ І

p(x10) – p(x8) =6-5=1= a(x8,x10) = 1, f(x8,x10) = 0 ∈ І

p(x11) – p(x8) = 7-5=2 = a(x8,x11) = 2, f(x8,x11) = 0 ∈ І

p(x13) – p(x8) = 7-5=2 = a(x8,x13) = 2, f(x8,x13) = 0 ∈ І

p(x13) – p(x10) = 7-6=1 ≠ a(x10,x13) = 3, f(x10,x13) = 0 ∈ N

p(x13) – p(x11) = 7-7=0 ≠ a(x11,x13) = 2, f(x11,x13) = 0 ∈ N

p(x14) – p(x12) = 7-7=0 ≠ a(x12,x14) = 2, f(x12,x14) = 0 ∈ N

P(x14) – p(x13) = 7-7 ≠ a(x13,x14) = 5, f(x13,x14) = 0 ∈ N  
Выполним результирующее окрашивание.

Максимальный поток найти не удается. Увеличим вершинные числа неокрашенных вершин на единицу.

p(x1) = 0 ; p(x2) = 2; p(x3) = p(x4)=3; p(x5)= p(x6) = 4

p(x7)=p(x8) = 5 p(x9) = p(x10)=6;

p(x11) = p(x12) = 7 p(x13) = p(x14) = 7+1=8

Повторяем шаг 2. Формируем множества I, R, N

p(x2) – p(x1) =2-0 = a(x1,x2) = 2, f(x1,x2) = 0 ∈ І

p(x3) – p(x1) = 3-0 = a(x1,x3) = 3, f(x1,x3) = 0 ∈ І

p(x4) – p(x2) =3-2=1 = a(x2,x4) = 1, f(x2,x4) = 0 ∈ І

p(x5) – p(x2) =3-2=1 = a(x2,x5) = 1, f(x2,x5) = 0 ∈ І

p(x7) – p(x2) =4-2=2 = a(x2,x7) = 2, f(x2,x7) = 0 ∈ І

p(x5) – p(x3) =4-3=1 = a(x3,x5) = 1, f(x3,x5) = 0 ∈ І

p(x6) – p(x3) =4-3=1= a(x3,x6) = 1, f(x3,x6) = 0 ∈ І

p(x8) – p(x3) = 5-3=2 = a(x3,x8) = 2, f(x3,x8) = 0 ∈ І

p(x7) – p(x4) = 4-3=1= a(x4,x7) = 1, f(x4,x7) = 0 ∈ І

p(x9) – p(x4) = 4-3=1 = a(x4,x9) = 1, f(x4,x9) = 0 ∈ І

p(x8) – p(x5) =4-3=1 = a(x5,x8) = 1, f(x5,x8) = 0 ∈ I

p(x10) – p(x5) =4-3=1= a(x8,x10) = 1, f(x8,x10) = 0 ∈ I

p(x9) – p(x7) = 6-5=1 = a(x7,x9) = 1, f(x7,x9) = 0 ∈ І

p(x10) – p(x7) = 6-5=1 = a(x7,x10) = 1, f(x7,x10) = 0 ∈ І

p(x12) – p(x7) =5-4=1 = a(x12,x7) = 1, f(x12,x7) = 0 ∈ I

p(x12) – p(x9) =5-4=1= a(x9,x12) = 1, f(x9,x12) = 0 ∈ I

p(x7) – p(x6) =5-4= 1 = a(x6,x7) = 1, f(x6,x7) = 0 ∈ І

p(x11) – p(x6) = 5-4=1= a(x6,x11) = 1, f(x6,x11) = 0 ∈ І

p(x10) – p(x8) =6-5=1= a(x8,x10) = 1, f(x8,x10) = 0 ∈ І

p(x11) – p(x8) = 7-5=2 = a(x8,x11) = 2, f(x8,x11) = 0 ∈ І

p(x13) – p(x8) = 7-5=2 = a(x8,x13) = 2, f(x8,x13) = 0 ∈ І

p(x13) – p(x10) = 8-6=2 ≠ a(x10,x13) = 3, f(x10,x13) = 0 ∈ N

p(x13) – p(x11) = 8-7=1 ≠ a(x11,x13) = 2, f(x11,x13) = 0 ∈ N

p(x14) – p(x12) = 8-7=1 ≠ a(x12,x14) = 2, f(x12,x14) = 0 ∈ N

P(x14) – p(x13) = 8-8 ≠ a(x13,x14) = 5, f(x13,x14) = 0 ∈ N  
Выполним результирующее окрашивание.

Максимальный поток найти не удается. Увеличим вершинные числа неокрашенных вершин на единицу.

p(x1) = 0 ; p(x2) = 2; p(x3) = p(x4)=3; p(x5)= p(x6) = 4

p(x7)=p(x8) = 5 p(x9) = p(x10)=6;

p(x11) = p(x12) = 7 p(x13) = p(x14) = 8+1=9

Повторяем шаг 2. Формируем множества I, R, N

p(x2) – p(x1) =2-0 = a(x1,x2) = 2, f(x1,x2) = 0 ∈ І

p(x3) – p(x1) = 3-0 = a(x1,x3) = 3, f(x1,x3) = 0 ∈ І

p(x4) – p(x2) =3-2=1 = a(x2,x4) = 1, f(x2,x4) = 0 ∈ І

p(x5) – p(x2) =3-2=1 = a(x2,x5) = 1, f(x2,x5) = 0 ∈ І

p(x7) – p(x2) =4-2=2 = a(x2,x7) = 2, f(x2,x7) = 0 ∈ І

p(x5) – p(x3) =4-3=1 = a(x3,x5) = 1, f(x3,x5) = 0 ∈ І

p(x6) – p(x3) =4-3=1= a(x3,x6) = 1, f(x3,x6) = 0 ∈ І

p(x8) – p(x3) = 5-3=2 = a(x3,x8) = 2, f(x3,x8) = 0 ∈ І

p(x7) – p(x4) = 4-3=1= a(x4,x7) = 1, f(x4,x7) = 0 ∈ І

p(x9) – p(x4) = 4-3=1 = a(x4,x9) = 1, f(x4,x9) = 0 ∈ І

p(x8) – p(x5) =4-3=1 = a(x5,x8) = 1, f(x5,x8) = 0 ∈ I

p(x10) – p(x5) =4-3=1= a(x8,x10) = 1, f(x8,x10) = 0 ∈ I

p(x9) – p(x7) = 6-5=1 = a(x7,x9) = 1, f(x7,x9) = 0 ∈ І

p(x10) – p(x7) = 6-5=1 = a(x7,x10) = 1, f(x7,x10) = 0 ∈ І

p(x12) – p(x7) =5-4=1 = a(x12,x7) = 1, f(x12,x7) = 0 ∈ I

p(x12) – p(x9) =5-4=1= a(x9,x12) = 1, f(x9,x12) = 0 ∈ I

p(x7) – p(x6) =5-4= 1 = a(x6,x7) = 1, f(x6,x7) = 0 ∈ І

p(x11) – p(x6) = 5-4=1= a(x6,x11) = 1, f(x6,x11) = 0 ∈ І

p(x10) – p(x8) =6-5=1= a(x8,x10) = 1, f(x8,x10) = 0 ∈ І

p(x11) – p(x8) = 7-5=2 = a(x8,x11) = 2, f(x8,x11) = 0 ∈ І

p(x13) – p(x8) = 7-5=2 = a(x8,x13) = 2, f(x8,x13) = 0 ∈ І

p(x13) – p(x10) = 9-6=3 = a(x10,x13) = 3, f(x10,x13) = 0 ∈ І

p(x13) – p(x11) = 9-7=2 = a(x11,x13) = 2, f(x11,x13) = 0 ∈ І

p(x14) – p(x12) = 9-7=2 = a(x12,x14) = 2, f(x12,x14) = 0 ∈ І

P(x14) – p(x13) = 9-9 ≠ a(x13,x14) = 5, f(x13,x14) = 0 ∈ N  
Выполним результирующее окрашивание.

Повторяем шаг 2. Формируем множества I, R, N

min{i(1, 2),i (2, 7),i (7, 12),i (12,14)}=min{5,2,1,4}=1;

f(1,2)=f(2, 7)=f(7, 12)=f(12,14)=0+1=1;

i(1, 2)=5-1=4; i(2, 7)=2-1=1; i(7, 12)=1-1=0; (R) i(12, 14)=4-1=4;

p(x1) = 0 ; p(x2) = 2; p(x3) = p(x4)=3; p(x5)= p(x6) = 4

p(x7)=p(x8) = 5 p(x9) = p(x10)=6;

p(x11) = p(x12) = 7 p(x13) = p(x14) = 8+1=9

min{i(1, 2),i (2, 4),i (4, 9),i (9, 12),i (12,14)}=min{4,2,3,4,3}=2;

f(1,2)=1+2=3; f(2, 4)=f(4,9)=f(9, 12)=0+2=2; f(12,14)=1+2=3;

i(1, 2)=4-2=2; i(2, 4)=2-2=0(R); i(4, 9)=3-2=1; i(9, 12)=4-2=2; i(12, 14)=3-2=1;

min{i(1, 2),i (2, 7),i (7, 9),i (9, 12),i (12,14)}=min{2,1,1,2,1}=1;

f(1,2)=3+1=4; f(2, 7)=1+1=2; f(7,9)=0+1=1; f(9, 12)=2+1=3; f(12,14)=3+1=4;

i(1, 2)=2-1=1; i(2, 7)=1-1=0 (R); i(7, 9)=1-1=0(R);

i(9, 12)=2-1=1; i(12, 14)=1-1=0(R);

min{i(1, 3),i (3, 8),i (8, 13),i (13,14)}=min{5,2,2,7}=2;

f(1, 3) = 0+2 = 2 f(3, 8) = 0+2 = 2 f(8, 13) = 0 + 2 = 2 f(13, 14) = 0+2 = 2

i(1, 3)=5-2=3; i(3, 8)=2-2=0 (R); i(8, 13)=2-2=0(R); i(13, 14)=7-2=5 ;

min{i(1, 2),i (2, 5),i (5, 10),i (10, 13),i (13,14)}=min{1,2,1,3,5}=1;

f(1,2)=4+1=5; f(2, 5)=0+1=1; f(5,10)=0+1=1; f(10, 13)=0+1=1; f(13,14)=2+1=3;

i(1, 2)=1-1=0(R); i(2, 5)=2-1=1; i(5, 10)=1-1=0(R);

 i(10, 13)=3-1=2; i(13, 14)=4-1=4;

Существует единствинный путь (1,2)(2,7)(7,12)(12,14), по котором мы можем попасть их Х1 в Х14. Все ребра на этом пути равны R, следоватильно, макимальный поток по минимальной стоимости найден.